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Correspondence principle for a brane in Minkowski space and vector mesons*

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Abstract: We consider a 3-brane in D-dimensional Minkowski space and discuss a correspondence principle analogous to the AdS/CFT correspondence. For a 3-brane of finite thickness in the transverse directions, we obtain a spectrum for vector gravitational perturbations which correspond to vector mesons. The spectrum agrees with the one obtained in truncated AdS space by de Téramond and Brodsky provided $D = 10$ and the bulk mass scale M is of order the geometric mean of the Planck mass ($\bar{M}$) on the brane and Λ_{QCD} ($M \sim (\bar{M}\Lambda_{QCD})^{1/2} \sim 10^9\,\mathrm{GeV}$).

Keywords: Brane Dynamics in Gauge Theories, p-branes, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

The AdS/CFT correspondence [1] is by now well-established, even though it has yet to be proved. It has led to novel approaches to gravity and an understanding of the hierarchy problem, raising the possibility of observing strong gravitational effects at TeV scales [2]. Moreover, it has served as a useful tool in analyzing Yang-Mills theories non-perturbatively [3]. Although the CFT is not phenomenologically relevant, significant progress was recently made toward extracting information related to experiments, following the work of Polchinski and Strassler [4]. They showed that by introducing an IR cutoff in AdS space, one reproduces the hard scattering behavior of QCD by convoluting the string amplitude (which exhibits soft high energy behavior) with AdS wavefunctions. Subsequent work showed that this model reproduces known theoretical and experimental results in hadronic physics pointing perhaps to a gravity dual of QCD [5 – 23].

Another interesting possibility is of the extra dimensions forming a flat space of large or even infinite volume [24 – 26]. In this case, light Kaluza-Klein modes may dominate even at low energies [27 – 29], leading to a modification of Newton's Law of gravity at astronomically large distances [30 – 38]. Reproducing the results on hardonic physics obtained in AdS space is a challenge in flat Minkowski space due to the lack of a holographic principle.

Here we make an attempt at understanding the spectrum of hadronic resonances using an extension of the Dvali-Gabadadze-Porrati (DGP) model [27, 28]. We consider a fat 3-brane of thickness $\sim 1/M$ in the transverse directions, where M is the D-dimensional

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bulk Planck mass. We solve the Einstein equations for vector gravitational perturbations and compare the spectrum with the masses of vector mesons. We obtain agreement with similar results in AdS space by de Téramond and Brodsky [20] for $D = 10$, provided the mass scale M is

$$M \sim \sqrt{\bar{M}\Lambda_{QCD}} \sim 10^9\,\text{GeV} \tag{1.1}$$

where $\bar{M}$ is the four-dimensional Planck mass. Unlike in AdS space, there is no need for an artificial IR cutoff; the finite width of the brane acts as a natural cutoff leading to normalizable solutions of the wave equation. We also discuss the conformal dimensions of the corresponding operators which create the hadronic resonances. Slicing the Minkowski space into hyperboloids, as advocated in [39, 40] (see also [41] for a related discussion of the conformal algebra on the brane), we obtain a correspondence principle which yields conformal dimensions in agreement with expectations.

Our discussion is organized as follows. In section 2 we show how a holographic screen is obtained in flat Minkowski space in analogy with AdS space [1]. Our discussion is an extension of of the discussion in [39] the novel element being the brane. In section 3 we consider the case of a scalar field. We solve the wave equation and discuss the conformal dimensions of the corresponding operators on the brane. In section 4 we consider gravitational perturbations. We derive the Einstein equations and solve them for vector gravitational perturbations which decouple. We obtain the spectrum of normalizable modes and compare it to similar results in truncated AdS space [20]. In section 5 we present our conclusions. Pertinent results in AdS space are summarized in appendix A.

2. Holography

Consider a flat 3-brane in D-dimensional Minkowski space parametrized by cartesian coordinates X^A $(A = 0, 1, \ldots, D-1)$. Coordinates in the brane will be denoted by x^μ $(\mu = 0, 1, 2, 3)$; transverse coordinates will be y^a $(a = 1, \ldots, D-4)$. The brane is assumed to be the hypersurface $\vec{y} = \vec{0}$. The D-dimensional metric is

$$ds^2 = dX^A dX_A = dx^\mu dx_\mu + d\vec{y}^2\,. \tag{2.1}$$

Let us foliate the space with hyperboloids [39]

$$X^A X_A = x^\mu x_\mu + \vec{y}^2 = -\mathbf{u}^2\,. \tag{2.2}$$

The light-cone surface $\mathbf{u} = 0$ splits the Minkowski space into three regions. A similar splitting occurs on the 3-brane. For definiteness, we shall concentrate on the region inside the future light cone foliated by euclidean AdS hyperboloids ($\mathbf{u}^2 > 0$, $x^0 > 0$). The other regions can be treated similarly. For a fixed $\mathbf{u} > 0$, standard AdS holographic arguments suggest that all information is encoded on the boundary which is a sphere of dimension $D-2$ (S^{D-2}). To see this in detail, parametrize the hyperboloid (2.2) with $\mathbf{u} > 0$ as

$$X^0 = \mathbf{u}\cosh\rho\,, \qquad \vec{X} = \mathbf{u}\sinh\rho\,\vec{\Omega}\,, \qquad (\vec{\Omega}^2 = 1) \tag{2.3}$$

where $X^A = (X^0, \vec{X})$. The Hadamard two-point function

$$G(X^A, X'^A) = \frac{\Gamma((D-2)/2)}{4\pi^{D/2}} \frac{1}{((X-X')^A(X-X')_A)^{(D-2)/2}} \tag{2.4}$$

may be expanded in terms of harmonic functions Ψ, where

$$\nabla^2 \Psi = 0 \,. \tag{2.5}$$

The laplacian in the coordinate system (2.3) reads

$$\nabla^2 = \frac{1}{\mathbf{u}^{D-1}} \partial_{\mathbf{u}}(\mathbf{u}^{D-1}\partial_{\mathbf{u}}) + \frac{1}{\mathbf{u}^2 \sinh^{D-2}\rho} \partial_\rho(\sinh^{D-2}\rho\, \partial_\rho) + \frac{1}{\mathbf{u}^2 \sinh^2\rho} \nabla^2_\Omega \,. \tag{2.6}$$

The eigenfunctions of ∇^2_Ω are spherical harmonics,

$$\nabla^2_\Omega Y_{\ell\vec{m}}(\vec{\Omega}) = -\ell(\ell+D-3) Y_{\ell\vec{m}}(\vec{\Omega}) \,. \tag{2.7}$$

The corresponding Hadamard function is a polynomial,

$$G^{(Y)}_\ell(\vec{\Omega}\cdot\vec{\Omega}') = \sum_{\vec{m}} Y^*_{\ell\vec{m}}(\vec{\Omega}) Y_{\ell\vec{m}}(\vec{\Omega}') = F\left(-\ell, \ell+D-3; \frac{D-2}{2}; \frac{1-\vec{\Omega}\cdot\vec{\Omega}'}{2}\right) . \tag{2.8}$$

A general harmonic function may be written as a product

$$\Psi_{\lambda\ell\vec{m}} = U_\lambda(\mathbf{u}) \mathsf{F}_{\lambda\ell}(\rho) Y_{\ell\vec{m}}(\vec{\Omega}) \tag{2.9}$$

where $\mathsf{F}_{\lambda\ell}(\rho)$ obeys

$$\frac{1}{\sinh^{D-2}\rho}(\sinh^{D-2}\rho \mathsf{F}'_{\lambda\ell})' - \frac{\ell(\ell+D-3)}{\sinh^2\rho} \mathsf{F}_{\lambda\ell} = -|\lambda|^2 \mathsf{F}_{\lambda\ell} \,, \qquad \lambda = \frac{D-2}{2} + i\xi \tag{2.10}$$

with solution

$$\mathsf{F}_{\lambda\ell}(\rho) = \sinh^\ell\rho\, F\left(\ell+\lambda, \ell+\lambda^*; \ell+\frac{D-1}{2}; \frac{1-\cosh\rho}{2}\right) . \tag{2.11}$$

Finally, the "radial" equation is

$$\frac{1}{\mathbf{u}^{D-1}} (\mathbf{u}^{D-1} U'_\lambda)' - \frac{|\lambda|^2}{\mathbf{u}^2} U_\lambda = 0 \tag{2.12}$$

whose solutions are $U_\lambda(\mathbf{u}) = \mathbf{u}^{-\lambda}, \mathbf{u}^{-\lambda^*}$.

The Hadamard function (2.4) may be expressed in terms of Hadamard functions on the AdS_{D-1} hyperboloids as

$$G(X^A, X'^A) = \int_{\frac{D-2}{2}-i\infty}^{\frac{D-2}{2}+i\infty} \frac{d\lambda}{2\pi i} \mathbf{u}^{-\lambda} \mathbf{u}'^{-\lambda^*} G_\lambda(\rho, \vec{\Omega}; \rho', \vec{\Omega}') \tag{2.13}$$

where

$$G_\lambda(\rho, \vec{\Omega}; \rho', \vec{\Omega}') = \sum_\ell G^{(Y)}_\ell(\vec{\Omega}\cdot\vec{\Omega}') \mathsf{F}_{\lambda\ell}(\rho) \mathsf{F}_{\lambda\ell}(\rho') \,. \tag{2.14}$$

It is most easily evaluated by noting that it is a function of the distance between two points on the unit hyperboloid ($\mathbf{u} = 1$) is

$$\mathsf{d}(\rho, \vec{\Omega}; \rho', \vec{\Omega}') = -2 - 2\cosh\rho\cosh\rho' + 2\sinh\rho\sinh\rho'\,\vec{\Omega}\cdot\vec{\Omega}'\,. \tag{2.15}$$

Choosing axes so that $\rho' = 0$, the resulting two-point function obeys eq. (2.10) with $\ell = 0$. We obtain from (2.11) with $\ell = 0$,

$$G_\lambda(\rho, \vec{\Omega}; \rho', \vec{\Omega}') = \frac{\Gamma(\lambda)\Gamma(\lambda^*)}{(4\pi)^{(D-1)/2}\Gamma((D-1)/2)}\, F\left(\lambda, \lambda^*; \frac{D-1}{2}; 1 + \frac{\mathsf{d}}{4}\right) \tag{2.16}$$

where we included an appropriate normalization factor [39]. For large distances ($\rho \to \infty$), standard hypergeometric function identities yield

$$G_\lambda(\mathsf{d}) \sim \frac{1}{(-\mathsf{d})^\lambda} + \text{c.c.} \tag{2.17}$$

with scaling dimensions λ, λ^*. This should correspond to two-point functions of operators of conformal dimensions λ, λ^* on the sphere S^{D-2} spanned by $\vec{\Omega}$ [39]. Thus, we obtain a holographic screen as $\rho \to \infty$ in analogy with AdS space.

Restricting attention to the brane at $\vec{y} = \vec{0}$, eq. (2.13) yields an expansion of a two-point function on the brane in terms of holographic data,

$$\frac{1}{((x-x')^2)^{(D-2)/2}} \sim \int_{\frac{D-2}{2}-i\infty}^{\frac{D-2}{2}+i\infty} \frac{d\lambda}{2\pi i}\, \Gamma(\lambda)\Gamma(\lambda^*)\, \mathbf{u}^{-\lambda}\mathbf{u}'^{-\lambda^*} F\left(\lambda, \lambda^*; \frac{D-1}{2}; 1 + \frac{\mathsf{d}}{4}\right) \tag{2.18}$$

where d is given by (2.15) with $\vec{\Omega}, \vec{\Omega}'$ along the brane and we omitted an overall normalization constant.

Other two-point functions on the brane may also be obtained from (2.13) by expanding in $(\vec{y} - \vec{y}')^2$. Isolating the terms of order L in the transverse to the brane distance $(\vec{y} - \vec{y}')^2$, we obtain the two-point function

$$\begin{aligned}\frac{1}{((x-x')^2)^{L+\frac{D-2}{2}}} \sim \int_{\frac{D-2}{2}-i\infty}^{\frac{D-2}{2}+i\infty} \frac{d\lambda}{2\pi i}\, |\Gamma(L+\lambda)|^2\, \mathbf{u}^{-L-\lambda}\mathbf{u}'^{-L-\lambda^*} \times \\ \times F\left(L+\lambda, L+\lambda^*; L + \frac{D-1}{2}; 1 + \frac{\mathsf{d}}{4}\right).\end{aligned} \tag{2.19}$$

This is a Green function for operators with fixed angular momentum quantum number L in the transverse to the brane directions. It may also be obtained by a partial-wave expansion of the Green function (2.4). For $L = 0$, we recover the Hadamard function (2.18) which may be extended to the entire D-dimensional Minkowski space.

3. Scalar field

Here we discuss the dynamics of a scalar field in the DGP model [27] consisting of a flat 3-brane residing in D-dimensional Minkowski space. Having understood the emergence of a holographic screen, we concentrate on the behavior of Green functions near the brane. To

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this end, it suffices to work with cartesian coordinates. First, we solve the wave equation in the bulk. Inclusion of the brane then leads to singular expressions which ought to be regulated. We do this by giving the brane a finite width in the transverse directions. We show that this affects the conformal dimensions of the operators corresponding to the solutions of the wave equation.

3.1 In the bulk

As in section 2, coordinates in the brane will be denoted by x^μ ($\mu = 0, 1, 2, 3$); transverse coordinates will be y^a ($a = 1, \ldots, D-4$) the brane being at $y^a = 0$. The action for a massless scalar field in the bulk is

$$S_{\text{bulk}} = M^{D-2} \int d^4x d^{D-4} y \partial_A \Phi \partial^A \Phi \tag{3.1}$$

where M is the D-dimensional Planck mass. After Fourier transforming along the brane, we obtain the D-dimensional bulk wave equation

$$(\nabla_y^2 + p^2)\Phi(p, \vec{y}) = 0\,. \tag{3.2}$$

To solve this, expand in harmonics,

$$\Phi(p, \vec{y}) = \sum_{L, \vec{m}_y} \Phi_L(p, y) Y_{L\vec{m}_y}(\vec{\Omega}_y)\,. \tag{3.3}$$

These spherical harmonics are defined on S^{D-5} spanned by $\vec{\Omega}_y$ and are part of the spherical harmonics considered in section 2 (eq. (2.7)) whose domain was S^{D-2} spanned by $\vec{\Omega}$. As we shall see, L matches the angular momentum quantum number of eq. (2.19).

For the Lth partial wave we have to solve the "radial" wave equation

$$\frac{1}{y^{D-5}}(y^{D-5}\Phi_L')' + p^2 \Phi_L - \frac{L(L+D-6)}{y^2}\Phi_L = 0 \tag{3.4}$$

whose well-behaved solution is

$$\Phi_L(p, y) = \mathcal{A}\, y^{-(D-6)/2} H_\alpha^{(1)}(py)\,, \qquad \alpha = L + \frac{D-6}{2} \tag{3.5}$$

where we included a normalization constant. This leads to the Green function

$$G(X^A, X'^A) = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-x')} \Phi_L^*(p, y)\Phi_L(p, y') \tag{3.6}$$

where $X^A = (x^\mu, y^a)$ and similarly for X'^A. After a Wick rotation, the integral may be calculated. In the limit $y, y' \to 0$ (approaching the brane), we obtain

$$G(X^A, X'^A) \sim \frac{(yy')^{L+2}}{((x-x')^2)^\Delta}\,, \qquad \Delta = 2 + \alpha = L + \frac{D-2}{2} \tag{3.7}$$

which (apart from the scaling factor $(yy')^{L+2}$) agrees with the two-point function derived in section 2 (eq. (2.19)). In the case $D = 10$, we have $\Delta = L + 4$, and the scaling agrees with the AdS result [42]. These solutions correspond to operators carrying SO(6) charge. We shall show that the scaling dimension is modified when the fluctuations of the brane in the transverse directions are properly accounted for.

The above result (3.7) may also be derived by projecting the D-dimensional Green function onto the Lth partial wave,

$$G(X^A, X'^A) \sim \int d^{D-5}\Omega_y \; Y_{L\vec{m}_y}(\vec{\Omega}_y) \frac{1}{((X-X')^2)^{(D-2)/2}} \tag{3.8}$$

where

$$(X-X')^2 = (x-x')^\mu (x-x')_\mu + (\vec{y}-\vec{y}')^2 \,. \tag{3.9}$$

3.2 On the brane

The above expressions ought to be regulated as we approach the brane in order to obtain quantities defined *on* the brane [28, 43]. This can be done by including fluctuations of the brane in the transverse directions [44–47]. We shall therefore assume that the brane has a finite transverse spread given by a uniform density $\sigma_\Lambda(y)$ ($y = |\vec{y}|$) where $1/\Lambda$ is the width and

$$\int d^{D-4}y \, \sigma_\Lambda(y) = 1 \,, \qquad \lim_{\Lambda\to\infty} \sigma_\Lambda = \delta^{D-4}(\vec{y}) \,. \tag{3.10}$$

The effects of the brane are governed by the action

$$S_{\rm brane} = \bar{M}^2 \int d^4x d^{D-4}y \sigma_\Lambda(y) \partial_\mu \Phi \partial^\mu \Phi \tag{3.11}$$

where $\bar{M}$ is the four-dimensional Planck mass and we assume

$$M \sim \Lambda \ll \bar{M} \sim 10^{19}\,\text{GeV} \,. \tag{3.12}$$

The wave eq. (3.2) is modified to

$$\left(M^{D-2}(p^2 + \nabla_y^2) + \bar{M}^2 p^2 \sigma_\Lambda(y)\right) \Phi(p, \vec{y}) = 0 \,. \tag{3.13}$$

For an explicit calculation, choose a constant density,

$$\sigma_\Lambda(y) = \frac{2\pi^{(D-4)/2}}{\Gamma((D-4)/2)} \Lambda^{D-4} \Theta\left(\frac{1}{\Lambda} - y\right) \,. \tag{3.14}$$

Outside the brane (in the bulk, $y > 1/\Lambda$), the wave equation reduces to eq. (3.4) for the Lth partial wave. The solution is given by eq. (3.5).

Inside the brane ($y < 1/\Lambda$), the solution can be written as

$$\Phi_L^{\rm brane}(p;y) = \mathcal{B}(y\Lambda)^{-(D-6)/2} J_\alpha(\kappa p y) \,, \qquad \kappa^2 = 1 + \frac{\sigma_\Lambda \bar{M}^2}{M^{D-2}} \,. \tag{3.15}$$

Matching expressions across the boundary ($y = 1/\Lambda$), we obtain

$$\begin{aligned} \mathcal{A} H_\alpha^{(1)}\left(\frac{p}{\Lambda}\right) &= \mathcal{B} J_\alpha\left(\frac{\kappa p}{\Lambda}\right) , \\ \mathcal{A} H_\alpha^{(1)\prime}\left(\frac{p}{\Lambda}\right) &= \kappa \mathcal{B} J'_\alpha\left(\frac{\kappa p}{\Lambda}\right) . \end{aligned} \tag{3.16}$$

For $p \ll \Lambda \sim M$, these can be written as

$$-\mathcal{A}\frac{i}{\pi}2^\alpha\Gamma(\alpha)\left(\frac{p}{\Lambda}\right)^{-\alpha} = \mathcal{B}J_\alpha\left(\frac{\kappa p}{\Lambda}\right),$$
$$\mathcal{A}\frac{i\alpha}{\pi}2^\alpha\Gamma(\alpha)\left(\frac{p}{\Lambda}\right)^{-\alpha-1} = \kappa\mathcal{B}J'_\alpha\left(\frac{\kappa p}{\Lambda}\right). \tag{3.17}$$

These conditions are compatible if

$$\left(\frac{\kappa p}{\Lambda}\right)J'_\alpha\left(\frac{\kappa p}{\Lambda}\right) + \alpha J_\alpha\left(\frac{\kappa p}{\Lambda}\right) = 0 \tag{3.18}$$

or, equivalently,

$$J_{\alpha-1}\left(\frac{\kappa p}{\Lambda}\right) = 0 \tag{3.19}$$

where we used the Bessel function identity

$$zJ'_\alpha(z) + \alpha J_\alpha(z) = zJ_{\alpha-1}(z)\,. \tag{3.20}$$

Another identity that will be useful later is

$$zJ'_\alpha(z) - \alpha J_\alpha(z) = -zJ_{\alpha+1}(z)\,. \tag{3.21}$$

Thus, we obtain the eigenvalues

$$p^2 = \frac{\beta^2_{\alpha-1,k}\Lambda^2}{\kappa^2}\,, \qquad \alpha - 1 = L + \frac{D-8}{2} \tag{3.22}$$

where $\beta_{\alpha-1,k}$ is the kth root of $J_{\alpha-1}$. For $\Lambda \sim M$, we have

$$p^2 \sim \beta^2_{\alpha-1,k}\frac{M^4}{\bar{M}^2} \tag{3.23}$$

so $p^2 \ll \Lambda^2 \sim M^2$ on account of $M \ll \bar{M}$, validating our approximations (unless we choose a root of the Bessel function of very high order).

To establish a correspondence principle, we ought to determine the conformal dimension of the operator on the brane which corresponds to these solutions of the wave equation (eq. (3.15)). In the momentum regime we are working ($p \ll \Lambda$), the tail of the wavefunction (3.5) in the bulk gives a negligible contribution. An examination of the scaling behavior of the Green function in the bulk led to a conformal dimension $\Delta = 2 + \alpha$ (eq. (3.7)). However, this conclusion did not take into account the effects of the finite width of the brane. We shall determine the conformal dimension by calculating the Green function on the brane. To this end, it is desirable to analytically continue the momentum beyond the discrete spectrum (3.22). Then the normalization constant $\mathcal{B}$ is fixed by going into the UV regime ($p^2 \gg \kappa^2/\Lambda^2 \sim \bar{M}^2/M^4$). Defining the inner product

$$\langle p|p'\rangle \equiv \frac{1}{2}\int_{y<1/\Lambda} d^{D-4}y\,\Phi_L^{\text{brane}}(p;y)\Phi_L^{\text{brane}}(p';y) \tag{3.24}$$

for the wavefunctions (3.15) on the brane, we may approximate it in the UV regime by

$$\langle p|p'\rangle \approx \frac{\pi^{(D-4)/2}}{\Gamma((D-4)/2)}\,\mathcal{B}(p)\mathcal{B}(p')\int_0^\infty dy\, y\, J_\alpha(\kappa p y)J_\alpha(\kappa p' y)\,. \tag{3.25}$$

Using the orthogonality property of Bessel functions,

$$\int_0^\infty dy\, y\, J_\alpha(qy)J_\alpha(q'y) = 2\delta(q^2-q'^2) \tag{3.26}$$

we deduce $\langle p|p'\rangle = \delta(p^2-p'^2)$, provided

$$\mathcal{B}^2 = \frac{2\pi^{(D-4)/2}}{\Gamma((D-4)/2)}\,\kappa^2 \sim \frac{\bar{M}^2}{M^2}\,. \tag{3.27}$$

Therefore the coefficient $\mathcal{B}$ is momentum-independent and may be ignored in our subsequent discussion.

For momenta satisfying $p \ll \Lambda$, the inner product of two wavefunctions (3.15) may be computed using standard manipulations of Bessel functions. Ignoring a momentum-independent overall factor, we obtain

$$\langle p|p'\rangle \sim \frac{\kappa p}{\Lambda\mathcal{C}(p,p')}J'_\alpha\left(\frac{\kappa p}{\Lambda}\right)J_\alpha\left(\frac{\kappa p'}{\Lambda}\right) - \frac{\kappa p'}{\Lambda\mathcal{C}(p,p')}J'_\alpha\left(\frac{\kappa p'}{\Lambda}\right)J_\alpha\left(\frac{\kappa p}{\Lambda}\right) \tag{3.28}$$

where $\mathcal{C}(p,p') = (\kappa p/\Lambda)^2 - (\kappa p'/\Lambda)^2$. Using the identity (3.20), we may write this as

$$\langle p|p'\rangle \sim \frac{\kappa p}{\Lambda\mathcal{C}(p,p')}J_{\alpha-1}\left(\frac{\kappa p}{\Lambda}\right)J_\alpha\left(\frac{\kappa p'}{\Lambda}\right) - \frac{\kappa p'}{\Lambda\mathcal{C}(p,p')}J_{\alpha-1}\left(\frac{\kappa p'}{\Lambda}\right)J_\alpha\left(\frac{\kappa p}{\Lambda}\right). \tag{3.29}$$

Evidently, for $p \neq p'$, we have $\langle p|p'\rangle = 0$ for eigenstates corresponding to the discrete spectrum (3.22) (obeying the boundary condition (3.19)). For the normalization of these states, we may use l'Hôpital's rule on the right-hand side of (3.29). In the limit $p' \to p$, we obtain

$$\langle p|p\rangle \sim \frac{1}{2}\left[J_{\alpha-1}\left(\frac{\kappa p}{\Lambda}\right)\right]^2 - \frac{\alpha\Lambda}{\kappa p}J_{\alpha-1}\left(\frac{\kappa p}{\Lambda}\right)J_\alpha\left(\frac{\kappa p}{\Lambda}\right) + \frac{1}{2}\left[J_\alpha\left(\frac{\kappa p}{\Lambda}\right)\right]^2 \tag{3.30}$$

which reduces to

$$\langle p|p\rangle \sim \frac{1}{2}\left[J_\alpha\left(\frac{\kappa p}{\Lambda}\right)\right]^2 \tag{3.31}$$

for momenta satisfying the boundary condition (3.19).

The Green function on the brane is defined by averaging the two-point function (3.6) over the transverse spread of the brane,

$$G^{\text{brane}}(x^\mu, x'^\mu) \equiv \int d^{D-4}y\,\sigma_\Lambda(y)G(x^\mu,\vec{y};x'^\mu,\vec{y})\,. \tag{3.32}$$

Using the wavefunctions on the brane (3.15), we may write this Green function in terms of the inner product (3.24) as

$$G^{\text{brane}}(x^\mu, x'^\mu) \sim \int \frac{d^4p}{(2\pi)^4}\,e^{ip\cdot(x-x')}\langle p|p\rangle\,. \tag{3.33}$$

In the small momentum (IR) limit, eq. (3.30) gives

$$\langle p|p\rangle \sim p^{2(\alpha-1)} \tag{3.34}$$

leading to the scaling behavior

$$G^{\rm brane}(x^\mu, x'^\mu) \sim \frac{1}{((x-x')^2)^\Delta}\,, \qquad \Delta = 1+\alpha \tag{3.35}$$

in disagreement with the bulk result $\Delta = 2+\alpha$ (eq. (3.7)). Notice that the bulk result is also obtained if we naïvely extrapolate (3.31), which is only valid for the discrete spectrum (3.22), to the IR regime. We have argued that the correct analytic continuation is provided by (3.30) and *not* (3.31), the latter being a special case of the former. The bulk result (3.7) does not lead to the correct correspondence principle for operators on the brane, unless the observable cannot "see" the brane (which may be the case with certain gravitational perturbations). In general, a calculation *on* the brane is needed for the analysis of conformal behavior.

4. Gravitational field

Next, we extend the discussion to gravity. Since we are interested in understanding the behavior of Green functions near the brane, cartesian coordinates will suffice. It is possible to extend the discussion of section 2 (see ref. [40]) and map our results to the holographic screen thus expressing the Green functions in terms of holographic data. We shall not do this here, as our main interest is in understanding the spectrum on the brane and its relation to vector mesons.

4.1 Field equations

As for a scalar, the action for the gravitational field also consists of a bulk term and a brane term. The bulk term is simply the D-dimensional Einstein-Hilbert action

$$S_{\rm bulk} = M^{D-2}\int d^DX\sqrt{-\det g}\,R^{(D)} \tag{4.1}$$

where g is the D-dimensional metric generating the D-dimensional Ricci scalar $R^{(D)}$.

We obtain the brane contribution to the action by dimensional reduction. Working with a flat brane at $\vec{y} = \vec{0}$, where $X^A = (x^\mu, \vec{y})$, we may decompose the metric as

$$ds^2 \equiv g_{AB}dx^A dx^B = \bar{g}_{\mu\nu}dx^\mu dx^\nu + \hat{g}_{ab}(dy^a + A^a_\mu dx^\mu)(dy^b + A^b_\mu dx^\mu)\,. \tag{4.2}$$

Assuming no dependence on the transverse directions, $\vec{y}$, the Ricci scalar may be written as [48]

$$\mathcal{R}^{(D)} = \bar{R}^{(4)} - \frac{1}{4}\partial_\mu(\ln\det\hat{g})\partial^\mu(\ln\det\hat{g}) - \frac{1}{4}\hat{g}^{ab}\hat{g}^{cd}\partial_\mu\hat{g}_{ac}\partial^\mu\hat{g}_{bd} - \frac{1}{4}\hat{g}_{ab}F^a_{\mu\nu}F^{b\mu\nu} \tag{4.3}$$

where $\bar{R}^{(4)}$ is the four-dimensional Ricci tensor generated by $\bar{g}_{\mu\nu}$ and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu\,. \tag{4.4}$$

The brane term is

$$S_{\text{brane}} = \bar{M}^2 \int d^4x d^{D-4}y \sqrt{-\det \bar{g}} \sigma_\Lambda(y) \mathcal{R}^{(D)} \tag{4.5}$$

where we included the effects of fluctuations in transverse directions given in terms of the profile function σ_Λ (eq. (3.10)).

Let us perturb around the Minkowski flat background

$$g_{AB} = \eta_{AB} + h_{AB} \tag{4.6}$$

Varying the action $S = S_{\text{bulk}} + S_{\text{brane}}$, given by eqs. (4.1) and (4.5), with respect to h_{AB}, we obtain the Einstein field equations. To solve these equations, in general, is challenging, because different gravitational perturbations are coupled to each other [45].

Here we concentrate on the fluctuations generated by the off-diagonal components of the metric (vector potential),

$$h_{a\mu} = A_{a\mu}\,. \tag{4.7}$$

We obtain the field equation

$$M^{D-2}(\partial_A\partial^A h_{a\mu} - \partial_\mu\partial^A h_{aA} - \partial_a\partial^A h_{\mu A} + \partial_\mu\partial_a h^A_A) + \bar{M}^2\sigma_\Lambda(y)\partial^\nu F_{a\mu\nu} = 0\,. \tag{4.8}$$

Choosing the harmonic gauge,

$$\partial^A h_{AB} = \frac{1}{2}\partial_B h^A_A \tag{4.9}$$

eq. (4.8) simplifies to

$$M^{D-2}\partial_A\partial^A A^a_\mu + \bar{M}^2\sigma_\Lambda(y)\partial^\nu F^a_{\mu\nu} = 0\,. \tag{4.10}$$

This field equation is consistent provided

$$\partial_\mu A^{a\mu} = 0 \tag{4.11}$$

(as for a massive vector field) in which case (4.10) reads

$$M^{D-2}\partial_A\partial^A A^a_\mu + \bar{M}^2\sigma_\Lambda(y)\Box A^a_\mu = 0\,. \tag{4.12}$$

Thus we obtained a wave equation for vector gravitational perturbations decoupled from all other modes. This equation is of the same form as the scalar wave equation (3.13). As we discussed in section 3, the field with angular momentum L in the transverse directions (SO$(D-4)$ quantum number) corresponds to an operator on the brane of conformal dimension

$$\Delta = 1 + \alpha = L + \frac{D-4}{2} \tag{4.13}$$

where we used eqs. (3.35) and (3.5). Moreover, the solution of the wave equation (4.12) leads to the discrete spectrum (3.22).

4.2 Vector mesons

A vector meson of internal angular momentum L has non-vanishing coupling to the interpolating operator [49, 50, 20]

$$\mathcal{O}^{\mu}_{L+3} = \bar{\psi}\gamma^{\mu} D_{\{\mu_1} \dots D_{\mu_L\}}\psi \tag{4.14}$$

where $D_\mu = \partial_\mu + ig_{YM}A_\mu$, and we have taken the traceless symmetric part of the tensor. This operator has dimension $\Delta = L + 3$.

Our earlier results on the correspondence between vector perturbations of the metric (4.7) and operators on the brane of conformal dimension given by (4.13) are not directly applicable to the interpolating operators (4.14). This is because the angular momentum quantum number appearing in (4.13) represents rotation in the transverse directions, whereas the operators in (4.14) have spin along the directions of the brane. In AdS space one obtains agreement in the conformal dimensions of these two types of operators, corresponding to strings spinning transversely and along the brane, respectively, in the limit of large dimensions with logarithmic corrections [51]. A similar argument should apply to our case of flat Minkowski space for $D = 10$.

Consequently, for $D = 10$ and large L, the interpolating operator (4.14) corresponds to the Lth partial wave solution of (4.12). The conformal dimension deduced earlier (eq. (4.13)) for $D = 10$ is $\Delta = L + 3$, in agreement with the QCD result for the operator (4.14). Moreover, the spectrum deduced from the wave equation (4.12), given by eq. (3.22) with $D = 10$,

$$p^2 = \frac{\beta^2_{L+1,k}\Lambda^2}{\kappa^2} \approx \beta^2_{L+1,k}\,\frac{M^4}{\omega_6\bar{M}^2} \tag{4.15}$$

where we used $\Lambda = M$ and eqs. (3.14) and (3.15). This coincides with the spectrum derived in truncated AdS space [20] provided we choose the ten-dimensional mass scale to be

$$M = (\pi^{3/2}\bar{M}\Lambda_{QCD})^{1/2} \sim 10^9\,\mathrm{GeV} \tag{4.16}$$

as advertised (eq. (1.1)). As was shown in [20], this spectrum is in good agreement with experimental results [52]. It should be noted that in our case, there was no need for an arbitrary IR cutoff. An effective cutoff was provided by the finite transverse width of the brane, $1/\Lambda \sim (\bar{M}\Lambda_{QCD})^{-1/2}$. We chose a crude profile for the brane (step function (3.14)). The prediction (4.15) of the spectrum may be improved upon by a refinement of the profile function (3.14).

5. Conclusions

In conclusion, we have discussed a correspondence principle for a 3-brane in D-dimensional flat Minkowski space in analogy with the AdS/CFT correspondence [1]. Following a suggestion in [39, 40], we foliated the flat Minkowski space with (EA)dS hyperboloids which led to a holographic screen. We were thus able to express the Green functions of interest near the brane in terms of holographic data. We realized a correspondence principle by considering fields with finite angular momentum in the transverse directions and showed that they led to a scaling behavior of the Green function in analogy with the results on the boundary propagator in AdS space [42]. We discussed a subtlety related to the correct analytic continuation of the spectrum of the D-dimensional wave equation to arbitrary momenta which affected the determination of the conformal dimension of the corresponding operator on the brane. Finally, we applied our results to the case of vector mesons which corresponded to vector gravitational perturbations in the bulk. We obtained a spectrum which was in agreement with AdS results [20] provided we chose $D = 10$ and the ten-dimensional mass scale as in (1.1).

Extending the discussion to other hadronic resonances entails a more complete solution of the ten-dimensional Einstein equations. Fermions may also be included by extending the model to supergravity. Work in this direction is in progress.

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A. Review of AdS

Here we review the pertinent features of AdS space for comparison with our results in flat Minkowski space. The wave equation for a massless scalar in $\mathrm{AdS}_5 \times S^5$ is [42]

$$\partial_z^2 \phi - \frac{3}{z}\partial_z \phi + p^2 \phi - \frac{L(L+4)}{z^2}\phi = 0 \tag{A.1}$$

where p^μ is the four-momentum on the boundary ($z \to 0$) and L is the S^5 angular momentum. The solution is

$$\phi(p; z) = \mathcal{B} z^2 J_\nu(pz)\,, \qquad \nu = L + 2\,. \tag{A.2}$$

The normalization constant is fixed by imposing the normalization condition

$$(p|p') \equiv \int_0^\infty \frac{dz}{z^3}\, \phi^*(p; z)\phi(p'; z) = \delta(p^2 - p'^2) \tag{A.3}$$

leading to a momentum-independent $\mathcal{B}$.

The propagator is given by

$$G(x, z; x', z') = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')}\phi^*(p; z)\phi(p; z') \tag{A.4}$$

from which one may deduce the boundary propagator in the limit $z, z' \to 0$. We obtain

$$\begin{aligned} G(x, z; x', z') &\sim (zz')^{\nu+2} \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')} p^{2\nu} \\ &\sim \frac{(zz')^\Delta}{((x-x')^2)^\Delta}\,, \end{aligned} \tag{A.5}$$

where $\Delta = \nu + 2 = L + 4$.

We wish to consider wavefunctions which live on the boundary but have a finite extent in the bulk direction z, $0 \leq z \leq z_0$ where z_0 is to be identified with the QCD scale ($z_0 = 1/\Lambda_{QCD}$) [20]. This cutoff z_0 breaks conformal invariance and can be thought of as providing a finite width for the brane residing on the boundary of AdS. Then CFT observables must be defined by averaging over the transverse width of the brane. Let us then define the boundary propagator by

$$\mathcal{G}(x,x') \sim \int_0^{z_0} \frac{dz}{z^3} G(x,z;x',z)\,. \tag{A.6}$$

The conformal limit is recovered in the IR ($pz_0 \ll 1$), because for small p, we have $J_\nu(pz) \sim (pz)^\nu$, therefore

$$\mathcal{G}(x,x') \sim z^{2(\nu+2)} \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-x')} p^{2\nu} \sim \frac{z^{2\Delta}}{((x-x')^2)^\Delta}\,, \tag{A.7}$$

in agreement with eq. (A.5).

If we impose the boundary condition

$$J_\nu(pz_0) = 0 \tag{A.8}$$

the momentum p is discretized,

$$p^2 = \frac{\beta_{\nu,k}^2}{z_0^2} \quad , \quad J_\nu(\beta_{\nu,k}) = 0\,. \tag{A.9}$$

Thus, $p \gtrsim o(1/z_0)$. These momenta no longer satisfy the requirement $pz_0 \ll 1$ and we are outside the conformal regime. Notice that for the discrete spectrum (A.9),

$$\int_0^{z_0} \frac{dz}{z^3} [\phi(p;z)]^2 = \frac{1}{2}[J_{\nu+1}(pz_0)]^2\,. \tag{A.10}$$

Naïvely analytically continuing this to low momenta, we obtain a behavior of $p^{2(\nu+1)}$ which yields the wrong boundary propagator (*cf.* eq. (A.7)). To do the analytic continuation correctly, we ought to first calculate the inner product

$$(p|p') = \int_0^{z_0} \frac{dz}{z^3} \phi(p;z)\phi(p';z) \tag{A.11}$$

which vanishes for p, p' obeying (A.9) with $p \neq p'$, and then take the limit $p' \to p$. This yields

$$(p|p) = \frac{1}{2}[J_\nu(pz_0)]^2 - \frac{\nu}{pz_0} J_\nu(pz_0) J_{\nu+1}(pz_0) + \frac{1}{2}[J_{\nu+1}(pz_0)]^2 \tag{A.12}$$

which reduces to (A.10) for momenta satisfying the quantization condition (A.8). Continuing this to low momenta, we obtain $(p|p) \sim p^{2\nu}$ and we recover eq. (A.7), as expected.

The discussion of the wave equation for a vector in AdS space is similar [53].

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064]; *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[3] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[4] J. Polchinski and M.J. Strassler, *Hard scattering and gauge/string duality*, *Phys. Rev. Lett.* **88** (2002) 031601 [hep-th/0109174].

[5] S.B. Giddings, *High energy QCD scattering, the shape of gravity on an IR brane and the froissart bound*, *Phys. Rev.* **D 67** (2003) 126001 [hep-th/0203004].

[6] J. Polchinski and M.J. Strassler, *Deep inelastic scattering and gauge/string duality*, *JHEP* **05** (2003) 012 [hep-th/0209211].

[7] A. Karch, E. Katz and N. Weiner, *Hadron masses and screening from AdS Wilson loops*, *Phys. Rev. Lett.* **90** (2003) 091601 [hep-th/0211107].

[8] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *JHEP* **07** (2003) 049 [hep-th/0304032].

[9] T. Sakai and J. Sonnenschein, *Probing flavored mesons of confining gauge theories by supergravity*, *JHEP* **09** (2003) 047 [hep-th/0305049].

[10] J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [hep-th/0306018].

[11] S.J. Brodsky and G.F. de Teramond, *Light-front hadron dynamics and AdS/CFT correspondence*, *Phys. Lett.* **B 582** (2004) 211 [hep-th/0310227].

[12] C. Núñez, A. Paredes and A.V. Ramallo, *Flavoring the gravity dual of $N = 1$ Yang-Mills with probes*, *JHEP* **12** (2003) 024 [hep-th/0311201].

[13] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Towards a holographic dual of large-N_c QCD*, *JHEP* **05** (2004) 041 [hep-th/0311270].

[14] S. Hong, S. Yoon and M.J. Strassler, *Quarkonium from the fifth dimension*, *JHEP* **04** (2004) 046 [hep-th/0312071].

[15] J. Erdmenger and I. Kirsch, *Mesons in gauge/gravity dual with large number of fundamental fields*, *JHEP* **12** (2004) 025 [hep-th/0408113].

[16] M. Kruczenski, L.A.P. Zayas, J. Sonnenschein and D. Vaman, *Regge trajectories for mesons in the holographic dual of large-N_c QCD*, *JHEP* **06** (2005) 046 [hep-th/0410035].

[17] K. Kang and H. Nastase, *High energy QCD from planckian scattering in AdS and the froissart bound*, hep-th/0410173.

[18] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, *Prog. Theor. Phys.* **113** (2005) 843 [hep-th/0412141].

[19] A. Paredes and P. Talavera, *Multiflavour excited mesons from the fifth dimension*, *Nucl. Phys.* **B 713** (2005) 438 [hep-th/0412260].

[20] G.F. de Teramond and S.J. Brodsky, *The hadronic spectrum of a holographic dual of QCD*, *Phys. Rev. Lett.* **94** (2005) 201601 [hep-th/0501022].

[21] K. Kang and H. Nastase, *Heisenberg saturation of the Froissart bound from AdS-CFT*, *Phys. Lett.* **B 624** (2005) 125 [hep-th/0501038].

[22] H. Nastase, *The soft pomeron from AdS-CFT*, hep-th/0501039.

[23] H. Nastase, *The rhic fireball as a dual black hole*, hep-th/0501068.

[24] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315].

[25] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[26] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344].

[27] G.R. Dvali, G. Gabadadze and M. Porrati, *4d gravity on a brane in 5D minkowski space*, *Phys. Lett.* **B 485** (2000) 208 [hep-th/0005016].

[28] G.R. Dvali and G. Gabadadze, *Gravity on a brane in infinite-volume extra space*, *Phys. Rev.* **D 63** (2001) 065007 [hep-th/0008054].

[29] M. Carena et al., *Brane effects on extra dimensional scenarios: a tale of two gravitons*, *Nucl. Phys.* **B 609** (2001) 499 [hep-ph/0102172].

[30] I.I. Kogan, S. Mouslopoulos, A. Papazoglou, G.G. Ross and J. Santiago, *A three three-brane universe: new phenomenology for the new millennium?*, *Nucl. Phys.* **B 584** (2000) 313 [hep-ph/9912552].

[31] E. Witten, *The cosmological constant from the viewpoint of string theory*, hep-ph/0002297.

[32] R. Gregory, V.A. Rubakov and S.M. Sibiryakov, *Opening up extra dimensions at ultra-large scales*, *Phys. Rev. Lett.* **84** (2000) 5928 [hep-th/0002072].

[33] C. Csáki, J. Erlich and T.J. Hollowood, *Quasi-localization of gravity by resonant modes*, *Phys. Rev. Lett.* **84** (2000) 5932 [hep-th/0002161].

[34] G.R. Dvali, G. Gabadadze and M. Porrati, *Metastable gravitons and infinite volume extra dimensions*, *Phys. Lett.* **B 484** (2000) 112 [hep-th/0002190].

[35] G.R. Dvali, G. Gabadadze and M. Porrati, *A comment on brane bending and ghosts in theories with infinite extra dimensions*, *Phys. Lett.* **B 484** (2000) 129 [hep-th/0003054].

[36] I.I. Kogan and G.G. Ross, *Brane universe and multigravity: modification of gravity at large and small distances*, *Phys. Lett.* **B 485** (2000) 255 [hep-th/0003074].

[37] C. Csáki, J. Erlich, T.J. Hollowood and J. Terning, *Holographic rg and cosmology in theories with quasi-localized gravity*, *Phys. Rev.* **D 63** (2001) 065019 [hep-th/0003076].

[38] I. Giannakis and H.-c. Ren, *Linearized analysis of the Dvali-Gabadadze-Porrati brane model*, *Phys. Lett.* **B 528** (2002) 133 [hep-th/0111127].

[39] J. de Boer and S.N. Solodukhin, *A holographic reduction of minkowski space-time*, *Nucl. Phys.* **B 665** (2003) 545 [hep-th/0303006].

[40] S.N. Solodukhin, *Reconstructing minkowski space-time*, hep-th/0405252.

[41] G. Siopsis, *Holography in the Penrose limit of AdS space*, *Phys. Lett.* **B 545** (2002) 169 [hep-th/0205302].

[42] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[43] G. Gabadadze and M. Shifman, *Softly massive gravity*, *Phys. Rev.* **D 69** (2004) 124032 [hep-th/0312289].

[44] E. Kiritsis, N. Tetradis and T.N. Tomaras, *Thick branes and 4D gravity*, *JHEP* **08** (2001) 012 [hep-th/0106050].

[45] C. Middleton and G. Siopsis, *Fat branes in infinite volume extra space*, hep-th/0210033.

[46] S.L. Dubovsky and V.A. Rubakov, *Brane-induced gravity in more than one extra dimensions: violation of equivalence principle and ghost*, *Phys. Rev.* **D 67** (2003) 104014 [hep-th/0212222].

[47] M. Kolanovic, M. Porrati and J.-W. Rombouts, *Regularization of brane induced gravity*, *Phys. Rev.* **D 68** (2003) 064018 [hep-th/0304148].

[48] J. Polchinski, *String theory, Vol. I*, Cambridge University Press, Cambridge 1998.

[49] D.J. Gross and F. Wilczek, *Asymptotically free gauge theories, 2*, *Phys. Rev.* **D 9** (1974) 980.

[50] H. Georgi and H.D. Politzer, *Electroproduction scaling in an asymptotically free theory of strong interactions*, *Phys. Rev.* **D 9** (1974) 416.

[51] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [hep-th/0204051].

[52] Particle Data Group collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.

[53] W. Muck and K.S. Viswanathan, *Conformal field theory correlators from classical field theory on anti-de Sitter spacem, II: vector and spinor fields*, *Phys. Rev.* **D 58** (1998) 106006 [hep-th/9805145].

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